International Journal of Research in Management & Social Science
Volume 13, Issue 4(III) October - December 2025
ISSN 2322 - 0899

WAITING TIME BEHIND THE SCENES: FIXED SERVICE SURRENDER TIME FOR THE FIRST CUSTOMER

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ABSTRACT

Service Surrender Facility of a normal queueing model is where a service at a particular queueing system can be submitted back after utilization or because it is of no use due to sometimes. When such a facility is available, it may be claimed by consumers initially deprived of it due to its unavailability. This may give rise to Secondary Queues which comprise of all such service takers.

This paper deals with the study of such a Service Surrender Facility in which the first customer has to wait for at least c units of time in the secondary queue before receiving the service.

Key Words: *Queue, Service Surrender Facility, Secondary Queues, Service Holding Time, Survival Function, Hazard Rate.*

Area of Research: Statistics, Operations Research

1: CONCEPT OF SERVICE SURRENDER FACILITY AND A SECONDARY QUEUE

The classical literature of queueing analysis studied only commonly observed primary queues. The results of the various queueing models depend on the input and the output distributions.

It has been observed in several situations that the customers not only have to wait in a primary queue but they also have to wait in the secondary queue. The analysis of bothe these queues will be different.

Secondary queues come into picture only when the system has a characteristic of 'SERVICE SURRENDER FACILITY'.

Sometimes, the service availed by the customers is returned by one or more customers due to dissatisfaction (e.g., misfit of the size), lack of requirement (e.g., railway or bus tickets) or at times may be because the availed service does not get exhausted even after utilization (e.g., library books). In such situations the SERVICE is said to be SURRENDERED. The availability of the service with the server is limited, and there will be a lot of demand for this surrendered services.

This paper mainly concentrates on the study of this secondary queue, which has not been carried out so far in the queueing literature but was first explained by *Kane Neela in 2001* as follows:

At a service counter with finite (say N) number of services available, after serving N customers, service will not be available for the subsequent customers. Instead when the $(N+1)^{th}$ customer arrives, the customer can now register his name in a 'waiting list'. These customers will be served as and when any of the previously served customer, surrenders the service. Here the waiting list will be another queue.

The time span for which the customer utilizes a service is a random variable.

Also, the customers on 'waiting list' do not know the amount of their waiting time or even whether will they get the service at all. Hence, a customer may register on 'waiting list' or quit the system.

SERVICE-SURRENDER FACILITY IS NOT AVAILABLE IN FOLLOWING SITUATIONS

1) A patient taking service from a doctor
2) A customer being served by a barber
3) Food ordered from and consumed in a restaurant

SERVICE-SURRENDER FACILITY IS AVAILABLE IN FOLLOWING SITUATIONS:

1) A customer possessing a railway reservation
2) A customer possessing a locker in the bank
3) Books borrowed from the library
4) Use of escalators / elevators / any public transport facility

1.1: PRIMARY QUEUE

A Primary Queue is the waiting line of the units at a service counter before the customers are registered on the 'waiting list'.

1.2 : SECONDARY QUEUE

A Secondary Queue is the waiting line of the customers, which are registered on the 'waiting list'.

- A customer can join the secondary queue only if after waiting through the primary queue.
- Secondary queue-analysis is an analysis of tge 'waiting time' of the customers.
- The customers of the secondary queue may have already waited for some time in the primary queue to find that the service is not available.
- These customers may get the service as and when the previously served customers surrender their services and this second phase of waiting time is unpredictable.

1.3: SERVICE HOLDING TIME (SHT)

Service Holding Time is the average duration of time for which a customer holds the service, before surrendering.

Though continuous, for the sake of analytical convenience SHT can be considered as a discrete random variable as described by (*Neela* Kane, 2004) in the following manner.

Let p = P(a customer enjoys the service for a unit time).

P(a customer surrenders the service sometime during the unit time) = (1-p) P(a customer completes 't' units of time, before he surrenders the service)

$= P(T = t) = p^{t-1}(1-p)$; $t = 1, 2, \ldots$ which is a Geometric distribution.

But 'p' will differ from customer to customer randomly, and can assume any value in the range [0, 1], it will also be a random variable.

The appropriate distribution of p is generally considered as Beta distribution with parameters

'a' and 'b' which is given by

$$f(p) = \frac{1}{\beta(a,b)} p^{a-1}(1-p)^{b-1} \quad ; a, b > 0;\ 0 < p < 1$$

$$= 0;\ \text{otherwise}$$

Now T is distributed as Geometric and p is distributed as Beta. Thus, according to *(Bartholomew, A multistage renewal process, 1963)*, SHT(T) will follow a compound Geometric Beta Distribution given by

$$P(T=t) = \frac{\beta(a+t-1, b+1)}{\beta(a,b)} \quad ; t = 1, 2 \ldots \text{ and } a, b > 0$$

$$= 0;\ \text{otherwise}$$

1.4: AVERAGE WAITING TIME OF A CUSTOMER IN THE SECONDARY QUEUE

The average duration of utilizing the service by a customer is given by the expectation of t

$$E(t) = \sum_{t=1}^{\infty} t f_t = \frac{a+b-1}{b-1} \quad ; \text{where } b \neq 1,\ b > a$$

1.5: THE DISTRIBUTION FUNCTION OF T

The Distribution function of T is given by

 ISSN 2322 - 0899

$$F_T = \sum_{t=1}^{T} f_t = 1- \frac{(a+b+T)}{b} f_{(T+1)}$$

1.6 : SURVIVAL FUNCTION

Let T > 0, have a density f_t and distribution function F_T. The survival function G_T is given by

$$G_T = \begin{cases} 1- F_T & \text{; if t is continuous} \\ 1- F_{T-1} & \text{;if is discrete} \end{cases}$$

Hence, for the above model,

$$G_T = \frac{(a+T-1+b)}{b} \times f_{(T)}$$

1.7 : HAZARD RATE / SERVICE WASTAGE

The Hazard Rate /Service Wastage S_T is given by

P{surrendering the service in (t, t+dt) /it was not surrendered till time T}

$$\therefore \text{Service Wastage } (S_T) = \frac{f_T}{G_T} \text{ where } G_T \text{ is the}$$

$$\therefore S_T = \frac{b}{(a+T-1+b)}$$

Service Wastage indicates the rate of surrender of the service.

Higher the value of service wastage S_T, higher is the Service Surrender Rate.

1.8: CONCLUSION

Graphs plotted for service wastage S_T against service holding time T for various values of a and b show that, for smaller values of parameter a, service wastage increases rapidly up to a time point nearer to the mean duration and gradually stabilizes for some time which can be explained as: the sharp decrease in service surrender may be due to various environmental and psychological factors and the sudden increase with elapse of large amount of time may be due to death of the customers, better availability of competitive facilities, etc.

2: MODEL WHERE THE FIRST CUSTOMER HAS TO WAIT FOR AT LEAST c UNITS OF TIME IN THE SECONDARY QUEUE BEFORE RECEIVING THE SERVICE

The model discussed in section 1 deals with the Service Holding Time (SHT), which is the average time for which the service is utilized, before surrendering.

The time for which the first customer has to wait for at least c units of time in the secondary queue before receiving the service may be the time for the server to freshen up or the time required to make the service ready for the customers in the secondary queue. The best examples of this situation are the waiting time for the customers before the flight boarding begins or the waiting time of the audience before a movie theatre entry begins.

Let X: Number of units of waiting time ~ Geometric(p) $P(X = x) = p^{x-1}(1-p)$; X = 1, 2, 3 …

= 0; otherwise

The distribution of the number of units of waiting time will now be

$P(X=x) = p^{x-1}(1-p)/p^{c-1}$; X = c, c+1, c+2 ……

= 0; otherwise which is a Truncated Geometric Distribution Therefore, the entire system is governed by Compound Truncated Geometric Beta Distribution.

Let T: Service Holding Time

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T ~ Compound Truncated Geometric Beta Distribution

$$P(T=t) = f_t = \int_0^1 f(p)\frac{p^{t-1}(1-p)}{p^{c-1}}\,dp$$

$$= \frac{b\,(a+t-c-1)^{(t-c)}}{(a+b+t-c)^{(t-c+1)}} \quad ; t = c, c+1, c+2,\ldots\ldots\ldots \quad ;a, b > 0$$

$$= 0 \qquad ; t < c$$

2.1: SERVICE HOLDING TIME (SHT)

$$P(T=t) = f_t = \frac{b\,(a+t-c-1)^{(t-c)}}{(a+b+t-c)^{(t-c+1)}}$$

2.2: EXPECTATION OF t

$$E(t) = \sum_{t=c}^{\infty} t f_t = \frac{c(b-1)+a}{(b-1)} \qquad ; b \neq 1$$

The Expected Service Hoding Time is independent of the time elapsed.

2.3: DISTRIBUTION FUNCTION OF T

$$F_T = \sum_{t=c}^{T} f_t = 1 - \frac{(a+T-c+b+1)}{b} \times f_{(T+1)}$$

2.4: SURVIVAL FUNCTION

Let T > 0, have a density f_t and distribution function F_T. The survival function G_T is given by

$$G_T = \begin{cases} 1 - F_T \;;\; if\ t\ is\ continuous \\ 1 - F_{T-1};\; if\ t\ is\ discrete \end{cases}$$

Hence, for the above model,

$$G_T = \frac{(a+T-c+b)}{b} \times f_{(T)}$$

2.5: HAZARD RATE / SERVICE WASTAGE

The Hazard Rate / Service Wastage

S_T = P{surrendering the service in (t, t+dt) / it was not surrendered till time T}

$$\therefore Service\ Wastage\ (S_T) = \frac{f_T}{G_T} = \frac{b}{(a+T-c+b)}$$

Higher the value of service wastage S_T, higher is the Service Surrender Rate.

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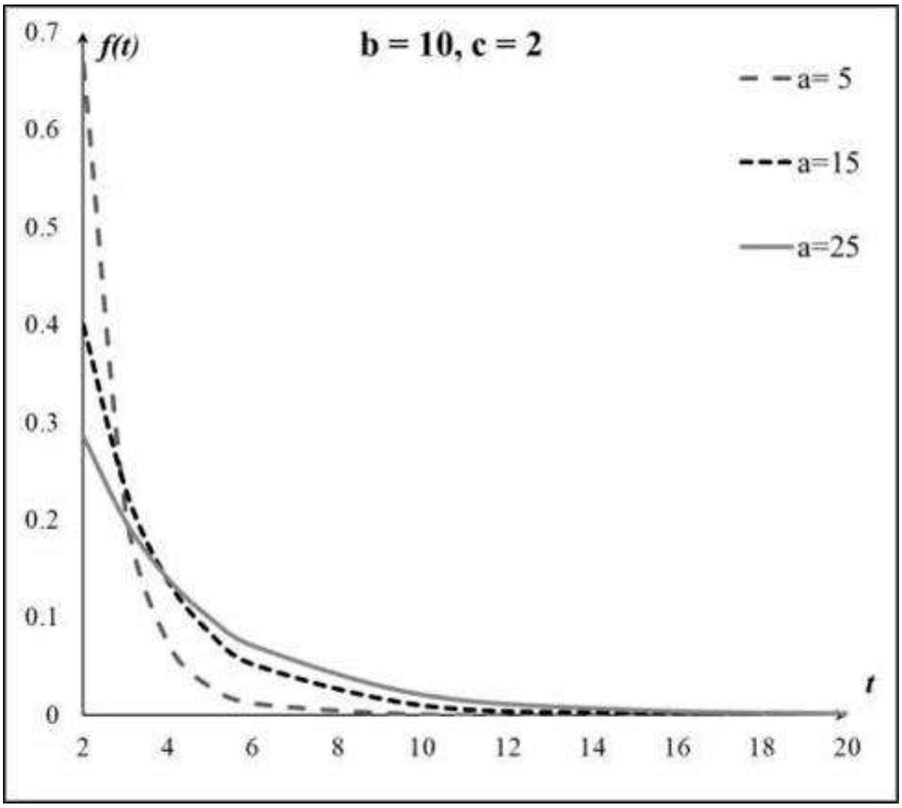
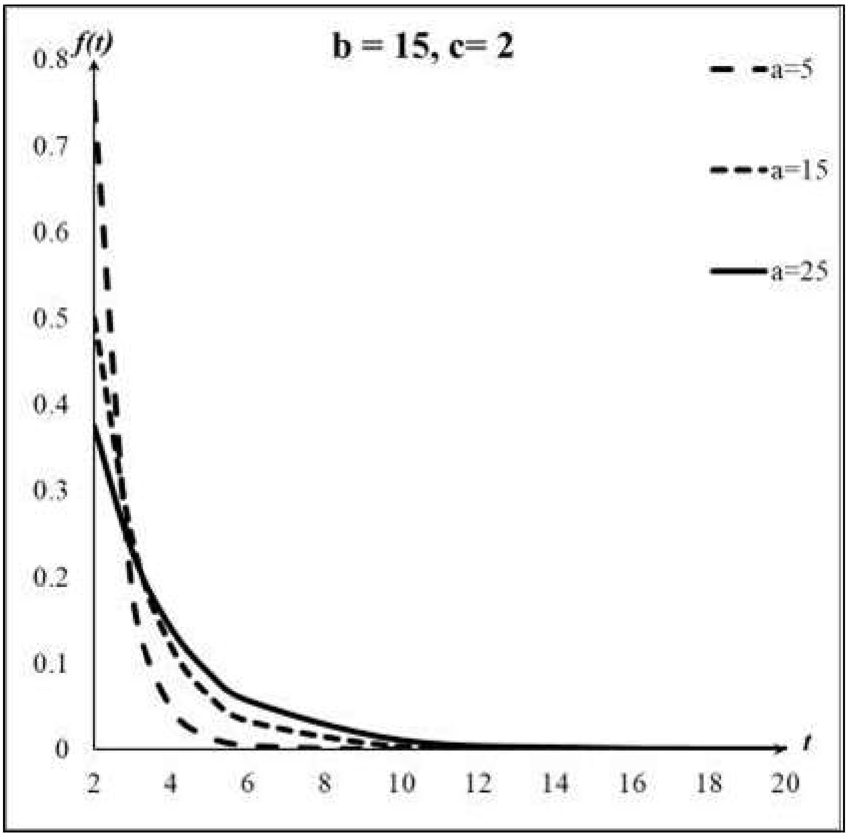
2.6 GRAPHS OF PROBABILITIY DISTRIBUTION OF SERVICE HOLDING TIME

Graph 1



Graph 2



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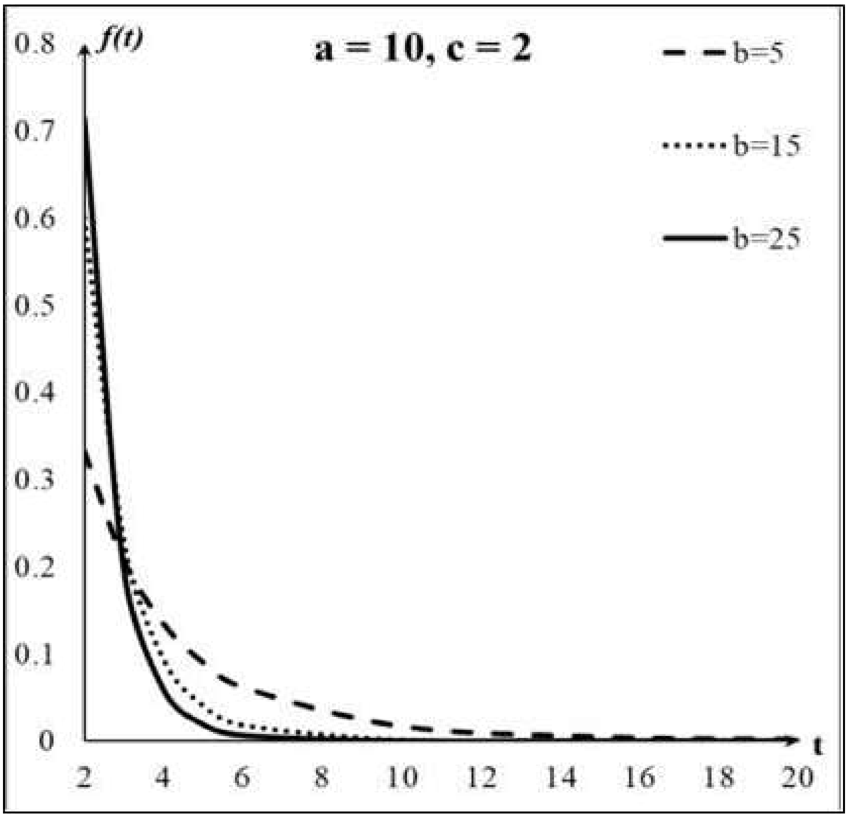
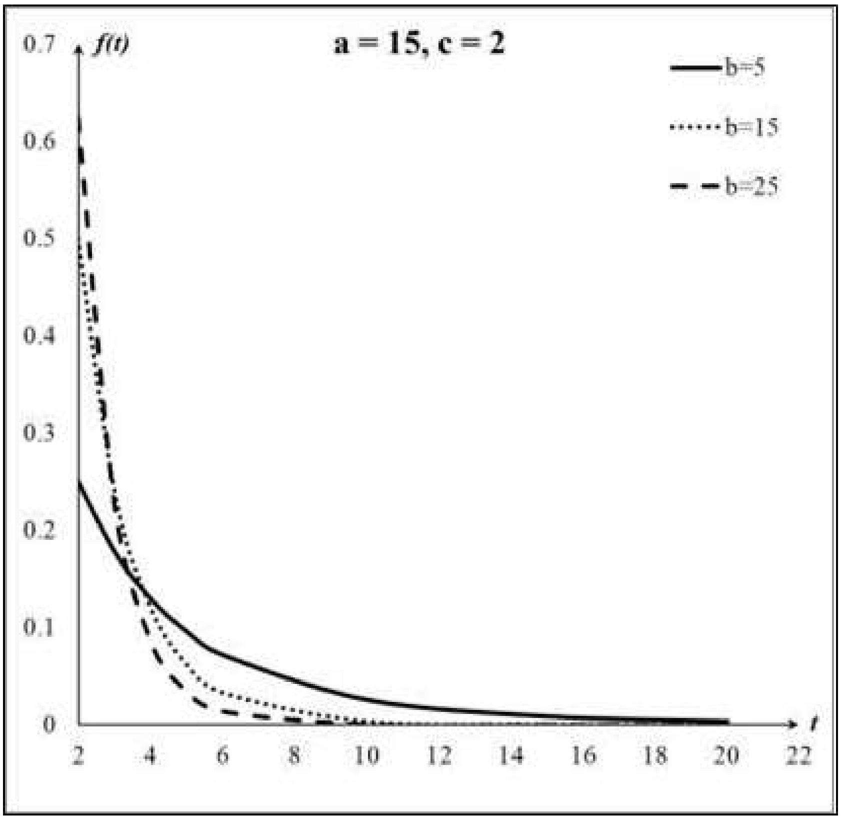
Graph 3



Graph 4



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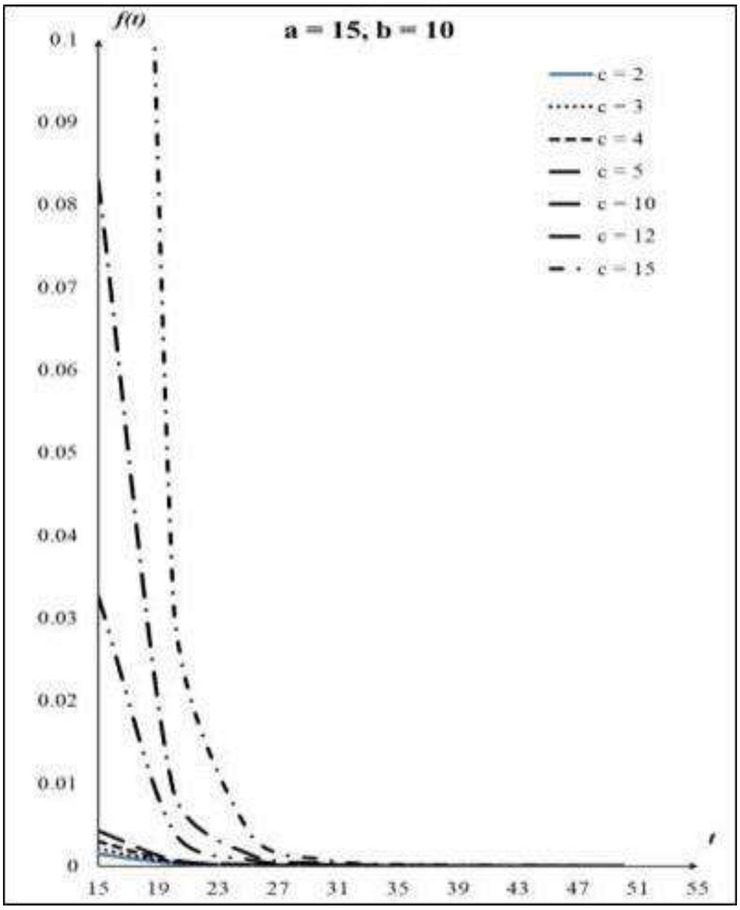
Graph 5

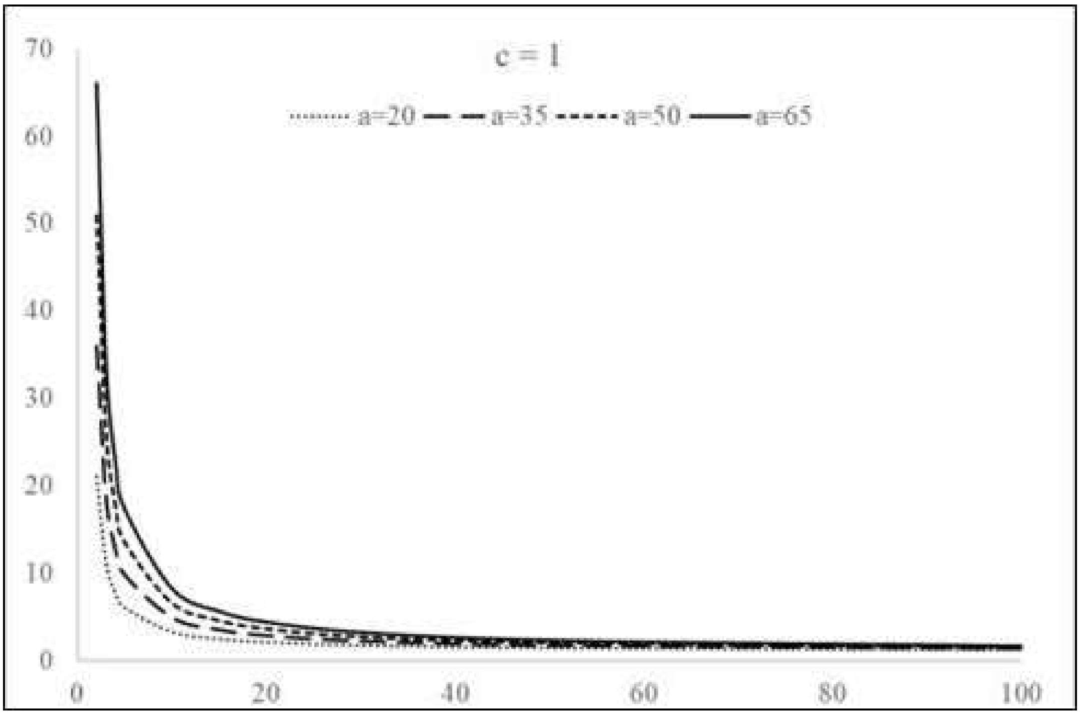


Graphs 1 and 2 show that for smaller values of a, the probability decreases rapidly as t increases whereas graphs 3 and 4 show that for higher values of b, the probability decreases rapidly as t increases.

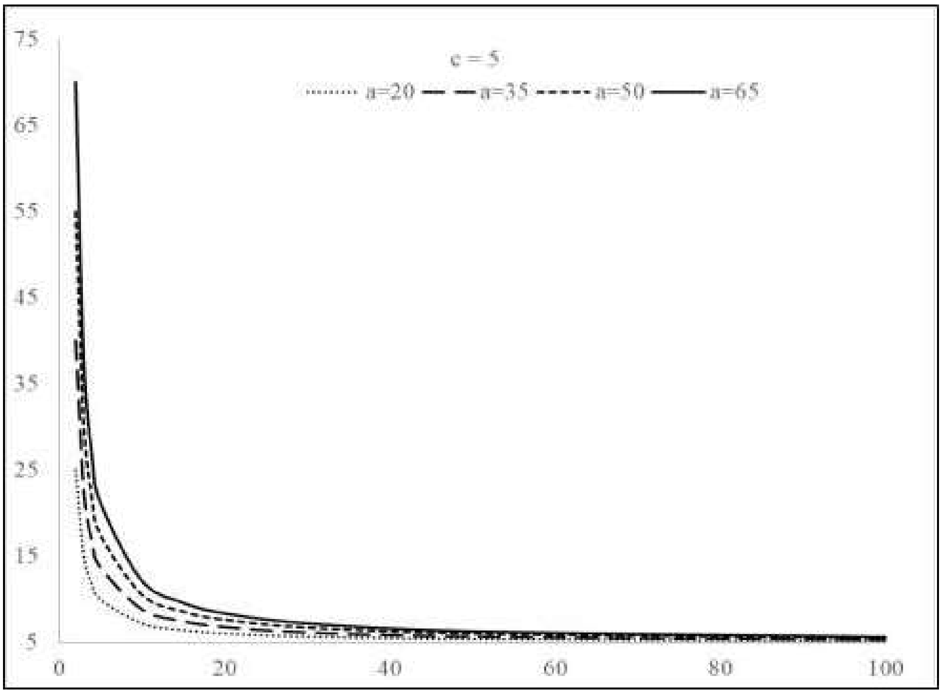
Graph 5 shows that when a and b are fixed, the probability decreases rapidly for higher values of c as t increases.

2.7 GRAPHS OF EXPECTED TIME

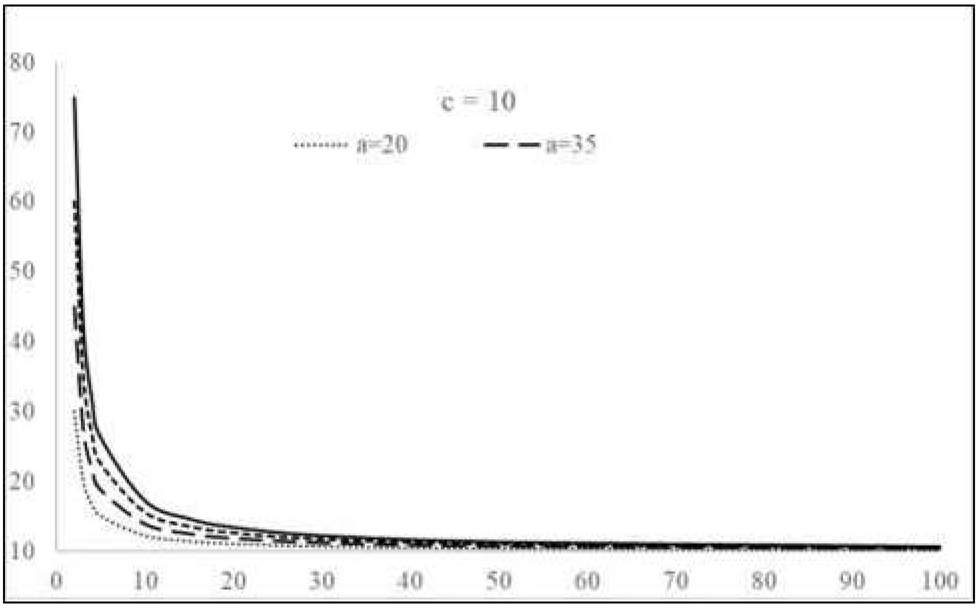
Graph 6



Graph 7



Graph 8



2.8: CONCLUSION

Graphs 6, 7 and 8 show that for various combinations of values of a, b and c, the Expected Service Holding Time becomes negligible with increase in values of a, b and c.

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